

Test 3 SOLUTIONS
Physics 2326 Section 001

Print your name (very legibly) on EACH PAGE of paper that you turn in including this test sheet. Staple all pages including the test sheet together. Be sure to right legibly for maximum credit.

Concept Questions (Answer all four—10 points each):

1. Consider two circular current loops of the same diameter each in the xy -plane, one directly above the other separated along their common central axis by a distance z . In what direction must the currents in each be flowing for there to be an attractive force between the two loops? Explain your answer.

The current must be flowing in the same direction, either clockwise or counterclockwise, in each loop for the force between the loops to be attractive.

2. The total magnet flux through any closed surface is always zero. Explain why.

The total flux through a surface is given by

$$\Phi = \oint \mathbf{B} \cdot d\mathbf{A}$$

That is a measure of the difference in flux entering and leaving the closed surface. Since field lines do not terminate, the number of lines entering the surface must equal the number of lines leaving the surface, so it must always be zero. This is an expression that there are no monopoles (on which field lines could terminate or originate).

3. Assume that a cosmic-ray proton is heading towards the Earth in a plane with the equator. In what direction will the proton be deflected (North, South, East, or West)? Justify your answer.

It will be deflected towards the East. That is because the force on the charged particle is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

In this case the magnetic field \mathbf{B} is directed towards the North, and the velocity \mathbf{v} is directed radially inward. The cross product will then be a vector pointing toward the East.

4. Why does a magnetic field tend to align magnetic dipoles in the same direction as the applied field?

The potential energy of a dipole $\vec{\mu}$ in a magnetic field \mathbf{B} is given by

$$U = -\vec{\mu} \cdot \mathbf{B}$$

That energy is minimized when the dipole moment and field are parallel (due to the minus sign).

Problems (Answer any four of the following five questions in any order but only the first 4 solutions will be graded—15 points each):

5. The timing device in an automotive wiper system is based on an RC time constant that utilizes a $0.500 \mu\text{F}$ capacitor and a variable resistor. Over what range must R be made vary to achieve time constants of 2.00 to 15.0 s?

Knowns:

$$2.00 \text{ s} \leq \tau \leq 15.0 \text{ s} \quad \text{time constant range}$$

Equations: The time constant is given by

$$\tau = RC \quad (5.1)$$

where the resistance and capacitance are given by R and C , respectively. We then have

$$\begin{aligned} 2.00 \text{ s} &\leq \tau \leq 15.0 \text{ s} \\ 2.00 \text{ s} &\leq RC \leq 15.0 \text{ s} \\ 2.00 \text{ s} &\leq R(0.500 \mu\text{F}) \leq 15.0 \text{ s} \quad (5.2) \\ \frac{2.00 \text{ s}}{0.500 \mu\text{F}} &\leq R \leq \frac{15.0 \text{ s}}{0.500 \mu\text{F}} \\ \boxed{4.00 \text{ M}\Omega \leq R \leq 30.0 \text{ M}\Omega} \end{aligned}$$

6. A proton moves at 7.50×10^7 m/s, perpendicular to a magnetic field. The field causes the proton to travel in a circular path with a radius of 0.800 m. What is the field strength?

Knowns:

$v = 7.50 \times 10^7$ m/s	speed of proton
$r = 0.800$ m	radius of the path
$m = 1.67 \times 10^{-27}$ kg	mass of proton
$q = 1.602 \times 10^{-19}$ C	charge on proton

Equations: The centripetal force on the proton is due to the magnetic force acting on the proton. Since it is a circular path they are equal so we have

$$\begin{aligned} F_{cent} &= F_B \\ m \frac{v^2}{r} &= qvB \quad (6.1) \\ B &= \frac{mv}{r} \end{aligned}$$

We then get

$$\begin{aligned} B &= \frac{(1.67 \times 10^{-27} \text{ kg})(7.50 \times 10^7 \text{ m/s})}{(1.602 \times 10^{-19} \text{ C})(0.800 \text{ m})} \quad (6.2) \\ &= \boxed{0.977 \text{ T}} \end{aligned}$$

7. What is the magnetic field strength needed on a 200-turn square loop which is 20.0 cm on a side to create a maximum torque of 300 N•m if the loop has a 25.0-A current flowing in it.?

Knowns:

$\tau = 300 \text{ N}\cdot\text{m}$	maximum torque on loop
$N = 200$	number of turns in the loop
$I = 25.0 \text{ A}$	current in loop
$l = 20.0 \text{ cm}$	length of edge of the loop

Equations: The magnetic moment of the loop is given by

$$\mu = IA \quad (7.1)$$

where A is the area of the loop. The torque from a single loop is given by

$$\vec{\tau} = \vec{\mu} \times \mathbf{B} \quad (7.2)$$

Thus, the torque from N loops will be

$$\vec{\tau}_{\text{tot}} = N\vec{\mu} \times \mathbf{B} \quad (7.3)$$

which will be maximum when they are perpendicular. The area will be

$$A = l^2 \quad (7.4)$$

Putting it all together we have

$$\begin{aligned} \tau_{\text{max}} &= NI^2 B \\ B &= \frac{\tau_{\text{max}}}{NI^2} \\ &= \frac{(300 \text{ N}\cdot\text{m})}{(200)(25.0 \text{ A})(0.200 \text{ m})^2} \\ &= \boxed{1.50 \text{ T}} \end{aligned} \quad (7.5)$$

8. Image that the Earth's magnetic field (which will be taken to be 7.00×10^{-5} T at sea level at the poles) were to fade away. How much current and in what direction would it be required to flow in a loop extending around the Earth's equator would be required to replace the field? (Use 6.37×10^6 m. for the radius of the Earth and assume no contributions from materials inside the earth.)

Knowns:

$$R_e = 6.37 \times 10^6 \text{ m}$$

radius of Earth

$$B_e = 7.00 \times 10^{-5} \text{ T}$$

Earth's magnetic field at the poles

Equations: The magnetic field along the axis of a current loop of radius R a distance x from the center of the loop is given by

$$B_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \quad (8.1)$$

In this case we have $x = R = R_e$. We then have

$$\begin{aligned} B_x &= \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \\ &= \frac{\mu_0 I R^2}{2(2R^2)^{3/2}} \\ &= \frac{\mu_0 I}{2R\sqrt{8}} \\ I &= \frac{2RB_x\sqrt{8}}{\mu_0} \\ &= \frac{(2\sqrt{8})(6.37 \times 10^6 \text{ m})(7.00 \times 10^{-5} \text{ T})}{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})} \\ &= \boxed{2.01 \times 10^9 \text{ A to the West}} \end{aligned} \quad (8.2)$$

where the direction comes from the right-hand rule.

9. Niobium metal becomes a superconductor when cooled to 9K. Its superconducting characteristics are destroyed when the surface magnetic field exceeds 0.100 T. What is the maximum current a 2.00-mm-diameter niobium wire can carry and remain superconducting in the absence of any external magnetic field?

Knowns:

$$\begin{array}{ll} d = 2.00 \text{ mm} = 0.002 \text{ m} & \text{diameter of the wire} \\ B = 0.100 \text{ T} & \text{maximum surface magnetic field} \\ \mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A} & \text{permeability of free space} \end{array}$$

Equations: From Example 28.8 in the text the magnetic field at a distance r outside of a conductor (or on its surface) carrying current I is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

Taking r at the surface of the conductor we get

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} \\ &= \frac{(2\pi)(1.00 \times 10^{-3} \text{ m})(0.100 \text{ T})}{4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}} \\ &= \boxed{500 \text{ A}} \end{aligned}$$

which is a huge amount of current for such a small diameter wire!