

Test 1 SOLUTIONS
Physics 2326 Section 001

Print your name (very legibly) on EACH PAGE of paper that you turn in including this test sheet. Staple all pages including the test sheet together. Be sure to right legibly for maximum credit.

Concept Questions (Answer all four—10 points each):

1. Consider the electric field due to a nonconducting infinite plane having a uniform charge density. Explain why the electric field does not depend on the distance from the plane in terms of spacing of the electric field lines.

Consider a section of a vertical sheet carrying charge $+1$ coulomb. It has some number of field lines pointing out from it horizontally to the right and left, all uniformly spaced. The lines have the same uniform spacing close to the sheet and far away, showing that the field has the same value at all distances.

2. Two identical conducting spheres labeled A and B have net charges $q_1 = 4 q_0 (>0)$ and $q_2 = -6 q_0$ respectively. A third conducting sphere identical to the others is initially uncharged. This third sphere is touched first to the first sphere and then to the second sphere before being removed from the area. The net charge on the second sphere will now be ____ $\times q_0$. Explain how you got that answer.

When the first and third spheres are brought into contact, they will both come to the same charge (the average of the two) which will be $2q_0$. When they are separated, they will both continue to have this same charge. When the third sphere is now brought into contact with the second sphere, they will again come to the same (average) charge, which this time is $\boxed{-2q_0}$.

3. You may have heard that one of the safer places to be during a lightning storm is inside a car. Why would that be the case?

Gauss's law predicts that excess charge on a conductor will reside on the surface of the conductor. If a car is left charged by a lightning strike, then that charge will remain on the outside of the car, not harming the occupants. It turns out that during the lightning strike, the current also remains on the outside of the conductor. Note that it is not necessarily safe to be in a fiberglass car or a convertible during a thunderstorm..

4. Consider two electric dipoles in empty space. Each dipole has zero net charge. Does an electric force exist between the dipoles—that is, can two objects with zero net charge exert electric forces on each other? If so, is the force one of attraction or repulsion? Explain.

In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

Problems (Answer any four of the following five questions in any order but only the first 4 solutions will be graded—15 points each):

5. An airplane is flying through a thundercloud at a height of 2,000. m. If a charge concentration of +40.0 C is above the plane at a height of 3,000. m within the cloud and a charge concentration of -40.0 C is at a height 1,000. m, what is the electric field at the aircraft?

Knowns:

$$\begin{array}{ll} q_1 = 40.0 \text{ C} & \text{charge at 3000 m} \\ q_2 = -40.0 \text{ C} & \text{charge at 1000 m} \\ h = 2000 \text{ m} & \text{height of the airplane} \end{array}$$

Formula: The electric field at a distance r from a point charge of magnitude q is given as

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector from the charge towards the field point for a positive charge, and in the reverse direction for a negative charge.

Solution: We then have for the contribution to the total field at the location of the aircraft from the positive charge concentration

$$\begin{aligned} \mathbf{E}_+ &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \\ &= (9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{+40.0 \text{ C}}{(1000 \text{ m})^2} (-\hat{\mathbf{j}}) \\ &= -3.60 \times 10^5 \hat{\mathbf{j}} \text{ N/C} \end{aligned}$$

For the negative charge concentration we have

$$\begin{aligned} \mathbf{E}_- &= \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{\mathbf{r}} \\ &= (9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{-40.0 \text{ C}}{(-1000 \text{ m})^2} (\hat{\mathbf{j}}) \\ &= -3.60 \times 10^5 \hat{\mathbf{j}} \text{ N/C} \end{aligned}$$

From the principle of superposition the total electric field at the aircraft position will be

$$\begin{aligned} \mathbf{E}_{tot} &= \mathbf{E}_+ + \mathbf{E}_- \\ &= \boxed{-7.20 \times 10^5 \hat{\mathbf{j}} \text{ N/C}} \end{aligned}$$

6. Near its surface, the earth has an electric field that points straight down and has a magnitude of about 130 N/C. If a basketball has a diameter of 24.2 cm and a mass of 0.625 kg, what is the magnitude and sign of the charge that must be placed on the basketball so it will float in the air above the ground?

Knowns:

$m = 0.625 \text{ kg}$	mass of basketball
$g = 9.80 \text{ m/s}^2$	acceleration due to gravity
$\mathbf{E} = -130 \hat{\mathbf{j}} \text{ N/C}$	field of earth (y-direction is up)

Equations:

Electric field force on the ball if the charge is q is

$$\mathbf{F}_e = q\mathbf{E}$$

The gravitational force on the ball of mass m is

$$\mathbf{F}_g = -mg\hat{\mathbf{j}}$$

Solution:

We want to find the value of q which will give

$$\begin{aligned}\mathbf{F}_e + \mathbf{F}_g &= 0 \\ q\mathbf{E} - mg\hat{\mathbf{j}} &= 0 \\ q &= \frac{mg}{E} \\ &= \frac{(0.625 \text{ kg})(9.80 \text{ m/s}^2)}{-130 \text{ N/C}} \\ &= \boxed{-0.0471 \text{ C}} = \boxed{-47.1 \text{ mC}}\end{aligned}$$

It is negatively charged so the electric force will be upwardly directed in order to oppose gravity.

7. A proton is projected in the positive x direction into a region of a uniform electric field $\mathbf{E} = -6.00 \times 10^5 \hat{\mathbf{i}} \text{ N/C}$ at $t = 0$. The proton travels 7.00 cm before coming to rest. Determine the proton's initial speed.

Knowns:

$m_p = 1.67 \times 10^{-27} \text{ kg}$	mass of proton
$q = e = 1.60 \times 10^{-19} \text{ C}$	charge on proton (equal to electron)
$\mathbf{E} = -6.00 \times 10^5 \hat{\mathbf{i}} \text{ N/C}$	electric field
$d = 7.00 \text{ cm}$	distance over which the acceleration occurs

Equations: The electric force \mathbf{F}_e on the proton due to the field will be given by

$$\mathbf{F} = q\mathbf{E}$$

We also have the dynamics equation which says if an acceleration a is applied over a distance d then the initial and final velocities, v_i and v_f , respectively, will be given by

$$v_f^2 = v_i^2 + 2ad$$

The force equation plus Newton's Second Law, $F = ma$, allows us to calculate the acceleration from the field as

$$F = ma = qE$$

$$a = \left(\frac{q}{m} \right) E$$

In the problem we must find the initial velocity given that after a distance d the final velocity will be zero. So we get from the dynamics equation

$$v_f^2 = -2ad$$

$$= -2 \left(\frac{q}{m} \right) Ed$$

$$v_i = \sqrt{-2 \left(\frac{q}{m} \right) Ed}$$

$$= \sqrt{-2 \left(\frac{1.60 \times 10^{-19} \text{ C}}{1.67 \times 10^{-27} \text{ kg}} \right) (-6.00 \times 10^5 \text{ N/C}) (7.00 \times 10^{-2} \text{ m})}$$

$$= \boxed{2.84 \times 10^6 \hat{\mathbf{i}} \text{ m/s}}$$

and it will be in the positive x direction.

8. On a clear, sunny day, a vertical electric field of about 130 N/C points down over flat ground. What is the surface charge density on the ground for these conditions?

Knowns:

$$E = 130 \text{ N/C} \quad \text{magnitude of electric field (points down)}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 \quad \text{permittivity of free space}$$

Equations: The field due to a conductor with a uniform charge density of σ is given as

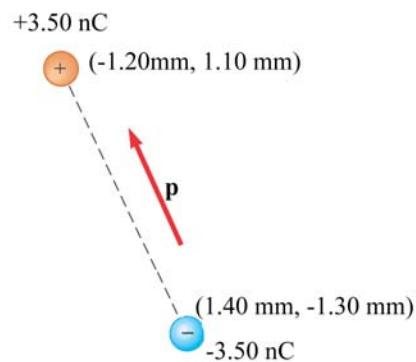
$$E = \frac{\sigma}{\epsilon_0}$$

giving the charge density in this case of

$$\begin{aligned} \sigma &= \epsilon_0 E \\ &= (8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(130 \text{ N/C}) \\ &= -1.15 \times 10^{-9} \text{ C/m}^2 = \boxed{-1.15 \text{ nC/m}^2} \end{aligned}$$

The minus sign comes from the fact that the field points down.

9. A small rigid object carries positive and negative 3.50-nC charges. It is oriented so that the positive charge has coordinates (-1.20 mm, 1.10mm) and the negative charge is at the point (-1.40 mm, -1.30 mm). The object is placed in an electric field $\mathbf{E} = (7,800\mathbf{i} - 4,900\mathbf{j})$ N/C. What is the potential energy of the object-field system when the object is in this orientation?



The knowns are given in the figure where \mathbf{p} is the dipole moment vector and it is all in an electric field of $\mathbf{E} = (7800\mathbf{i} - 4900\mathbf{j})$ N/C

Equations: The potential energy of a dipole in an electric field is given by

$$U = -\mathbf{p} \cdot \mathbf{E}$$

Solution: The dipole moment is the product of the charge times the distance between the charges, d . For d we have

$$\begin{aligned} \mathbf{d} &= (-1.20\mathbf{i} + 1.10\mathbf{j})\text{mm} - (1.40\mathbf{i} - 1.30\mathbf{j})\text{mm} \\ &= (-2.60\mathbf{i} + 2.40\mathbf{j})\text{mm} = (-2.60\mathbf{i} + 2.40\mathbf{j}) \times 10^{-3} \text{ m} \\ \mathbf{p} &= q\mathbf{d} \\ &= (3.50 \times 10^{-9} \text{ C})(-2.60\mathbf{i} + 2.40\mathbf{j}) \times 10^{-3} \text{ m} \\ &= (-9.10\mathbf{i} + 8.40\mathbf{j}) \times 10^{-12} \text{ C}\cdot\text{m} \end{aligned}$$

So for the potential energy we get

$$\begin{aligned} U &= -\mathbf{p} \cdot \mathbf{E} \\ &= -\left[(-9.10\mathbf{i} + 8.40\mathbf{j}) \times 10^{-12} \text{ C}\cdot\text{m}\right] \cdot \left[(7800\mathbf{i} - 4900\mathbf{j}) \text{ N/C}\right] \\ &= -(-7.098 - 4.116) \times 10^{-8} \text{ J} \\ &= 1.12 \times 10^{-7} \text{ J} = \boxed{112 \text{ nJ}} \end{aligned}$$